COSMO 2103 Exercise – Smith: [Simulating multirate adaptation and viscoelastic primitives]

1. For the single-state autoregressive trial-to-trial model of error-dependent learn discussed in class:

   \[
   e(n) = F(n) - x(n) \\
   x(n+1) = Ax(n) + Be(n)
   \]

   (a) A is usually close to (but less than) 1 (>0.7) and B is usually small (<0.3) and positive. Simulate the step response of this error-dependent learning model for a couple different value pairs for A & B.

   (b) *Optional* - Derive the asymptotic learning level \( x_\infty / F_\infty = B / (1 - A + B) \) and the time constant for learning in terms of the model parameters, A & B.

   (c) *Optional* - For what values of A & B is this error-dependent learning rule stable (in the BIBO sense)? Sketch the region of stability on an A-B axis.

2. Consider the parallel two-state (second order) version of this learning rule.

   (a) Simulate the step response (120 trials) for this learning rule.

   \[
   e(n) = F(n) - x(n) = F(n) - [xf(n) + xs(n)] \\
   xf(n+1) = Af*xf(n) + Bf e(n) \\
   xs(n+1) = As*xs(n) + Bs e(n)
   \]

   Let:

   \[
   F(0)=0; \ F(n)=1 \ for \ n>0. \\
   Af=0.6, \ Bf=0.2, \ As=0.99, \ Bs=0.02.
   \]

   (b) Simulate savings (rapid relearning).

   (c) Simulate Spontaneous recovery in a block of following learning and unlearning Error-Clamp trials.

   (d) *Optional* – [Difficult] How does the amount of spontaneous recovery depend on the parameters \{Af, Bf, As, & Bs\}.

   (b) *Optional* – Derive the asymptotic learning level for this learning rule as a function of the parameters for the 2\textsuperscript{nd} order and versions of this learning rule.

4. Simulate the first order gradient descent learning rule in Sing et al 2009, where motor adaptation is posited to result from a population of motor primitives with positively correlated responses to the position and velocity of motion. In particular, simulate the adaptive responses shown in panel 3C of that paper.