Linear Systems Theory
in Sensorimotor Control

(see Syllabus JvO, sections 1.1-1.5)
Example: Transfer Characteristic of the Low-Pass RC-Filter:
(= Exercise 10)

Model:

Frequency characteristic:

\[
G(\omega) = \frac{1}{\sqrt{1 + (\omega T)^2}}
\]

\[
\Phi(\omega) = -\arctan(\omega T)
\]

\[
h(\tau) = \frac{1}{RC} \exp\left(-\frac{\tau}{RC}\right)
\]

\[
s(t) = [1 - \exp\left(-\frac{t}{RC}\right)]
\]

\[T \equiv RC\]

TIME-CONSTANT
Example: Transfer Characteristic of the High-Pass RC-Filter: (Exercise 11)

Note: High Pass = ALL Pass - Low Pass
Transfer Characteristic of the Integrator: (Exercise 12)

\[ y(t) = \int_0^t x(\tau) \, d\tau \]

Questions:
- Show that this is a linear system
- What is the impulse response?
- What is the step response?
- Take \( x(t) = A \sin (2\pi f t) \) as input and the amplitude and phase for \( y(t) \) (N.B. these depend on \( f \!\)!
- Why is the LP system called ‘leaky integrator’?
Transfer Characteristic of the Differentiator: (= Exercise 13)

\[ y(t) = \frac{dx}{dt} \]

Questions:
- Show that this is a linear system
- What is the impulse response?
- What is the step response?
- Take \( x(t) = A \sin (2\pi f t) \) as input and the amplitude and phase for \( y(t) \)
Transfer Characteristic of the pure Delay: (= Exercise 14)

\[ y(t) = x(t - T) \]

Questions:
• Show that this is a linear system
• What is the impulse response?
• What is the step response?
• Take \( x(t) = A \sin (2\pi f t) \) as input and the amplitude and phase for \( y(t) \)
Feedback in linear systems:

\[
H \equiv \frac{Y}{X} = \frac{H1}{1 + H1 \cdot H2} \approx \frac{1}{H2}
\]
Example of a simple feedback system: the **smooth pursuit eye movement system**

(= Exercise 17)

**Question:** How can we determine $G$?

$$H \equiv \frac{\dot{E}}{\dot{T}} = \frac{G}{1 + G}$$

$$G = \frac{H}{1 - H}$$