Linear Systems Theory
in SensoriMotor Control

(see Syllabus JvO, sections 1.1-1.5)
Examples: The Vestibular-Ocular Reflex (VOR) and Saccades

- The saccade
  - VOR = step response of the oculomotor system to a STEP-change in head velocity
  - Saccade = step response of the oculomotor system to a STEP-change in target position

Quick phases (‘reset’ saccades) + Slow phase (VOR) = nystagmus
Example: Neural control of the eye plant: the simplest model

Response of OMN:

\[ R_m(t) = R_0 + k \cdot E(t) + r \cdot \frac{dE(t)}{dt} \], i.e.

\[ R_m(t) - R_0 \equiv \Delta R_m(t) = k \cdot E(t) + r \cdot \frac{dE(t)}{dt} \]

Impulse response:

\[ E(t) = E_0 \cdot \exp\left(-\frac{t}{T}\right) \]

with \( T = \frac{r}{k} \), the plant time constant
Frequency behaviour of the plant: its transfer characteristic

\[ G(\omega) = \frac{1}{\sqrt{k^2 + (r\omega)^2}} \]

\[ \Phi(\omega) = -\arctan\left(\frac{r\omega}{k}\right) \]
Extended example of the oculomotor plant (2nd order LP filter)

Plant
(globe+fat+eye muscles)

Low-pass (=slow) Linear System

\[ x(t) \rightarrow h_1(\tau) \rightarrow u(t) \rightarrow h_2(\tau) \rightarrow y(t) \]

See Exercises answers for time-domain analysis
LINEARITY: Superposition!

E(\omega) = H(\omega) \cdot M(\omega)

M(\omega) = \frac{E(\omega)}{H(\omega)}

One can reconstruct the motorneuron activity from measured eye movements and plant model.
Reconstruction of the OMN output in practice:

Comparison of normal saccades to saccades after an intravenous injection of diazepam (Valium).
Activity of a single neuron that innervates the right eye muscle of the right eye during a saccadic (rapid) eye movement.

During every saccade, more than 100000 motor neurons will act in this way....
Example: The vestibular organ (balance/head-in-space)
Output: activity of the vestibular nerve
the semicircular canals operate in three ‘antagonistic pairs’:

Lateral canals
Posterior canals
Anterior canals

L-R
LA-RP
RA-LP

(see Purves Ch. 13)
Activity (number of action potentials/s) of a primary vestibular nerve fiber, in response to sudden increase and decrease in head rotational velocity (‘step response’).

Note the exponential-like response of the neuron.
the VOR to a constant head rotation:
the **Step response** of the VOR:
Is the VOR a linear system? Does superposition apply?
Use of linearity of the VOR: how to rotate such that the slow-phase = constant?

(= Exercise 18)

Required stimulus:
the VOR has a high-pass characteristic:

\[
G(\omega) = \frac{\omega T}{\sqrt{1 + (\omega T)^2}}
\]

\[
\Phi(\omega) = \arctan\left(\frac{1}{\omega T}\right)
\]